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Book Review: *An Introduction to Chaos in Nonequilibrium Statistical Mechanics Cambridge*

An Introduction to Chaos in Nonequilibrium Statistical Mechanics. J. R. Dorfman, Cambridge Lecture Notes in Physics 14, Cambridge University 1999.

This book is based on a series of lectures given at the University of Utrecht in 1994 for fourth year students of physics. It describes fundamental bases of nonequilibrium statistical mechanics starting with the Boltzmann ergodic hypothesis and Gibbs mixing hypothesis, and later describes modern nonequilibrium statistical mechanics and its conceptual foundations. It gives a good introduction to modern research in transport theory which relates macroscopic properties of large systems to underlying microscopic dynamics. The book does not pretend to be mathematically rigorous but presents an extremely readable account of the conceptual foundations of nonequilibrium statistical mechanics. It does give, however, very good directions for those who require a deeper understanding of the underlying mathematical development. Several of the chapters also contain illuminating and relatively simple exercises.

I was quite taken by the book's emphasis on conceptual foundations and do not regret its lack of mathematical rigor. I am sure many physicists working in statistical mechanics will be delighted to learn how far the study of its foundations, especially the relation between microscopic reversibility and the macroscopic "arrow of time" has progressed. Recent research described in the book provides a number of interesting and deep connections between a system's microscopic dynamics, as revealed by its Lyapunov exponents that tell us how nearby trajectories behave in the course of time and transport properties, e.g., transport coefficients exemplified by a diffusion constant or conductivity. The important questions here are why such a connection should exist and the conditions required to ensure their validity. Dorfman provides marvelously illuminating answers to some of these questions based on simple model systems exemplified by the baker transform and the Arnold's cat map.

The organization of the book is as follows: Chapters 2–6 provide background required to understand standard methods of nonequilibrium statistical mechanics. Boltzmann's equation features prominently in the book. The Green–Kubo time correlation function formalism with its virtues and pitfalls and the fundamentals of classical chaos theory are introduced in a pedestrian manner. The baker transform and the Arnold cat map serve as toy models of microscopic dynamics and transport theory. One can obviously go much further in analyzing these simplified models than more physically realistic systems. Both models are defined in a two-dimensional phase space. The projection of their full phase space distribution function onto a one-dimensional subspace reveals a very important property of approach to equilibrium: the projected distribution function approaches equilibrium on a much shorter scale, set by the appropriate Lyapunov exponent, than does the full distribution function. The latter one has a time scale set by the characteristic times of ergodic and mixing behaviors. It is fundamental to realize that the two distribution functions are not characterized by the same time scales.

At this point it is not clear how general is the property that projected distributions, obtained by integrating out all but one of the variables from the complete Boltzmann equation, irreversibly approaches an equilibrium distribution. Dorfman proves this property for the baker's transformation but the same arguments pose difficulties for more realistic systems. Obviously this type of behavior originates from a fundamental dynamical instability of dynamic systems with positive Lyapunov exponents, in which a small change in the initial conditions leads to exponential divergence of the trajectories.

Chapters 7–18 discuss recent developments in the research on transport theory and especially the nonequilibrium steady states. Two approaches to transport theory–escape rate formalism for transport coefficients and the Gaussian thermostat method for transport coefficients are addressed in detail. These methods allow one to derive the detailed connection between macroscopic transport quantities such as the transport coefficients and the microscopic dynamical quantities such as the Lyapunov exponents and the Kolmogorov–Sinai entropies. The beautiful and simple relation between electrical conductivity and Lyapunov exponents is derived in detail. A similar connection also holds between the diffusion coefficients and microscopic quantities describing the microscopic chaotic motion of trajectories. The insight offered by these results is indeed fundamental.

Chapter 17, in particular, puts all the arguments for the emergence of irreversibility in nonequilibrium gases into a coherent framework in terms of the dynamical foundations of the Boltzmann equation. Again, the insight gained is that projected or reduced distribution functions can approach the equilibrium values on much shorter time scales than those needed for ergodicity and mixing of the system in phase space. The details of this irreversible behavior depend on the properties of the microscopic dynamics as well as the fact that a typical gas is composed of many particles. Though the book goes a long way towards solving the problem of how to derive the Boltzmann equation from first principles based the chaotic nature inherent in microscopic dynamics and a passage to the limit of a large number of particles, the issue is still not fully resolved. It is not too much to say that the Boltzmmann equation provides a bridge between microscopic and macroscopic chaos.

The Boltzmann equation obviously plays a fundamental role in Dorfman's book and is prominent at both its beginning and its end. In this respect the book comes full circle. It starts with the Boltzmann equation to compute transport properties of dilute gases. The positivity of at least some of the Lyapunov exponents, and therefore certain special properties of microscopic dynamical behavior, is needed to justify the Boltzmann equation approach to irreversible behavior itself. But also in its turn one uses the Boltzmann equation to compute the Lyapunov exponents and show that indeed some of them are positive,as required for the whole approach to be a sensible one. This constitutes a deep and pleasing consistency in the theory. One needs positivity of the Lyapunov exponents to justify the Boltzmann equation approach which, in turn provides tools to compute them and prove that indeed some of them are positive. To summarize, this is a very well written and readable book by one of the experts in the field. Its emphasis on conceptual developments, illustrated by simple dynamical models provides intereresting reading not just for specialists, but also for a more general physical audience seeking a better understanding of the current status of the conceptual foundations of nonequilibrium statistical mechanics.

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